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Title: Singularities of Base Polynomials and Gau-Wu Numbers

Abstract: In 2013, Gau and Wu introduced a unitary invariant, denoted by $k(A)$, of an $n \times n$ matrix A, which counts the maximal number of orthonormal vectors $\mathbf{x}_j$ such that the scalar products $\langle A\mathbf{x}_j, \mathbf{x}_j \rangle$ lie on the boundary of the numerical range $W(A)$. We refer to $k(A)$ as the Gau–Wu number of the matrix A. We write H_1 and iH_2 for the Hermitian and skew-Hermitian parts of A (respectively), and use them to define the base polynomial $F(x, y, t) = \det(xH_1 + yH_2 + tI)$. In this talk we will take an algebraic-geometric approach and consider the effect of the singularities of the base curve $F(x : y : t) = 0$, whose dual is the boundary generating curve, to classify $k(A)$. This continues the work of Wang and Wu classifying the Gau-Wu numbers for 3×3 matrices. Our focus on singularities is inspired by Chien and Nakazato, who classified $W(A)$ for 4×4 unitarily irreducible A with irreducible base curve according to singularities of that curve. When A is a unitarily irreducible $n \times n$ matrix, we give necessary conditions for $k(A) = 2$, characterize $k(A) = n$, and apply these results to the case of unitarily irreducible 4×4 matrices.

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