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Title: Geometry of finite Blaschke products: pentagons and pentagrams

Abstract: In this talk, I treat two types of curves induced by Blaschke product. For a Blaschke product B of degree d and λ on the unit circle, let ℓ_λ be the set of lines joining each distinct two preimages in $B^{-1}(\lambda)$. The envelope of the family of lines $\{\ell_\lambda\}_{\lambda\in\partial\mathbb{D}}$ is called the *interior curve* associated with B. In 2002, Daepp, Gorkin, and Mortini proved the interior curve associated with a Blaschke product of degree 3 forms an ellipse.

While let L_λ be the set of lines tangent to the unit circle at the d preimages $B^{-1}(\lambda)$ and consider the trace of the intersection points of each two elements in L_λ as λ ranges over the unit circle. This trace is called the *exterior curve* associated with B. The exterior curve associated with a Blaschke product of degree 3 forms a non-degenerate conic.

In this talk, I explain the existence of a duality-like geometrical property lies between the interior curve and the exterior curve. Using this property, I create some examples of Blaschke products whose interior curves consist of two ellipses.

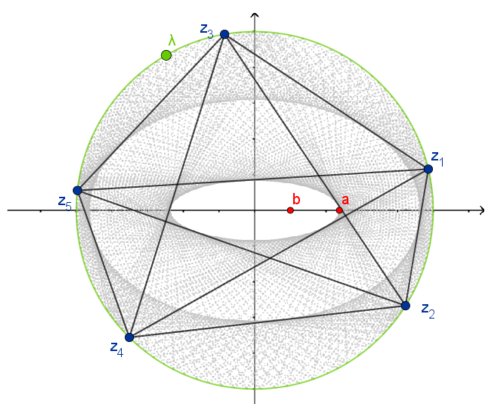

Figure 1: The envelope indicates the interior curve of $B(z) = z\frac{z^2-0.47}{1-0.47z^2}\frac{z^2-0.2}{1-0.2z^2}$. The interior curve consists of two ellipses, one is inscribed in the family of pentagons and the other is inscribed in the family of pentagrams.